**Part I**

1. Suppose that two balanced dice are rolled, and let X denote the absolute value of the difference between the two numbers that appear. Determine and sketch the p.f. of X.
2. Suppose that a box contains seven red balls and three blue balls. If five balls are selected at random, without replacement, determine the p.f. of the number of red balls that will be obtained.
3. If 10 percent of the balls in a certain box are red, and if 20 balls are selected from the box at random, with replacement, what is the probability that more than three red balls will be obtained?
4. Suppose that the number of hours X for which a machine will operate before it fails has a continuous distribution with p.d.f. f (x). Suppose that at the time at which the machine begins operating you must decide when you will return to inspect it. If you return before the machine has failed, you incur a cost of b dollars for having wasted an inspection. If you return after the machine has failed, you incur a cost of c dollars per hour for the length of time during which the machine was not operating after its failure. What is the optimal number of hours to wait before you return for inspection in order to minimize your expected cost?
5. Suppose that a random variable X has the uniform distribution on the interval [−2, 8]. Find the p.d.f. of X and the value of Pr(0 < X < 7).
6. An ice cream seller takes 20 gallons of ice cream in her truck each day. Let X stand for the number of gallons that she sells. The probability is 0.1 that X = 20. If she doesn’t sell all 20 gallons, the distribution of X follows a continuous distribution with a p.d.f. of the form cx for 0<x<20, 0 otherwise. Where c is a constant that makes Pr(X < 20) = 0.9. Find the constant c so that Pr(X < 20) = 0.9 as described above.
7. A civil engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that X = x is proportional to (x+1)(8−x) for x=0,...,7 (the possible values of X).
   1. Find the p.f of X.
   2. Find the probability that X will be at least 5.
8. Suppose that the p.d.f. of a random variable X is as follows:

f(x) = x/8 if 0<=x<=4, 0 otherwise

1. Find the value of t such that Pr(X<=t) = 1/4.
2. Find the value of t such that Pr(X>=t) = 1/2.

**Part 2**

**Problem 1**. The MIT soccer team has 2 games scheduled for one weekend. It has a 0.4 probability of not losing the first game. and a 0.7 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie. independent of what happens in the other game. The MIT team will receive 2 points for a win, 1 for a tie. and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend.

**Problem 2**. You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately by using the Poisson PMF. (For simplicity. exclude birthdays on February 29.)

**Problem 3**. Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws. the match is declared drawn. Each game is won by Fischer with probability 0.4. is won by Spassky with probability 0.3. and is a draw with probability 0.3. independent of previous games.

(a) What is the probability that Fischer wins the match?

(b) What is the PMF of the duration of the match?

**Problem 4**. An internet service provider uses 50 modems to serve the needs of 1000 customers. It is estimated that at a given time. each customer will need a connection with probability 0.01, independent of the other customers.

(a) What is the PMF of the number of modems in use at the given time?

(b) Repeat part (a) by approximating the PMF of the number of customers that need a connection with a Poisson PMF.

(c) What is the probability that there are more customers needing a connection than there are modems? Provide an exact. as well as an approximate formula based on the Poisson approximation of part (b).

**Problem 5**. A packet communication system consists of a buffer that stores packets from some source, and a communication line that retrieves packets from the buffer and transmits them to a receiver. The system operates in time-slot pairs. In the first slot, the system stores a number of packets that are generated by the source according to a Poisson PMF with parameter 'x; however, the maximum number of packets that can be stored is a given integer b, and packets arriving to a full buffer are discarded. In the second slot, the system transmits either all the stored packets or c packets (whichever is less). Here, c is a given integer with 0 < c < b.

1. Assuming that at the beginning of the first slot the buffer is empty, find the PMF of the number of packets stored at the end of the first slot and at the end of the second slot.
2. What is the probability that some packets get discarded during the first slot?

**Problem 6.** The Celtics and the Lakers are set to play a playoff series of n basketball games, where n is odd. The Celtics have a probability p of winning any one game, independent of other games.

(a) Find the values of p for which n = 5 is better for the Celtics than n = 3

(b) Generalize part (a) i.e. for any k > 0, find the values of p for which n = 2k+1 is better for the Celtics than n = 2k-1.

**Problem 7**. You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical. so to open the front door, you try them at random.

(a) Find the PMF of the number of trials you will need to open the door, under the following alternative assumptions: (1) after an unsuccessful trial. you mark the corresponding key. so that you never try it again. and (2) at each trial you are equally likely to choose any key.   
(b) Repeat part (a) for the case where the realtor gave you an extra duplicate key for each of the 5 doors.